A novel observation-driven model for the conditional median of double-bounded time series

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Abbreviated abstract: A new generalized autoregressive score (GAS) model is introduced. It is defined from the assumption that the conditional median of the Kumaraswamy distribution is a time-varying parameter under the GAS framework and pioneers the conditional median approach to analyze double-bounded time series from the GAS specification. We conduct simulation studies and present an empirical application.

Related publications:

– D. Creal *et al*, Journal of Applied Econometrics 28 (5), 777-795 (2013)
– P. Gorgi et al, Journal of Econometrics In Press, (2021)

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The GAS Framework and Previous Works

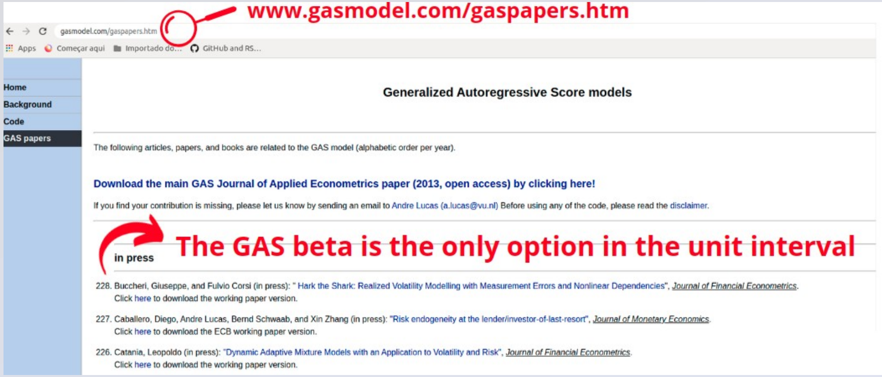
- Introduced by Creal *et al* (2013), the GAS framework considers a sequence of random variables of interest, $\tilde{y}_1, \ldots, \tilde{y}_n$, each one conditionally distributed as

$$\tilde{y}_t \sim p\left(\tilde{y}_t \vee f_t, F_{t-1}; \theta\right), t=1, \ldots, n,$$

- The updating equation for f_t is

$$f_t = \omega + \sum A_i s_{t-i} + \sum B_j f_{t-j},$$

- The updating mechanism considers that $s_t = S_t \nabla_t$, where $\nabla_t = \partial \log\left(p\left(y_t \vee f_t, F_{t-1}; \theta\right) / \partial f_t\right)$ is the score function corresponding to f_t and S_t is a scale factor, which is usually taken from the inverse of the information matrix corresponding to the updating mechanism.



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Kumaraswamy autoregressive score model (Kw-GAS)

- The Kw-GAS model is defined from the assumption that the conditional median of the Kw distribution is a time-varying parameter under the GAS framework.
- Thus, the novel observation-driven model for the conditional median of double-bounded time series is fully specified by the following equations:
  - Density:

$$p\left(\widetilde{y}_t \vee f_t, F_{t-1}; \theta\right) = \frac{\varphi \log(0.5)}{\log\left(1-\mu_t^{\varphi}\right)} y_t^{\varphi-1}\left(1-y_t^{\varphi}\right)^{\log(0.5)/\log\left(1-\mu_t^{\varphi}\right)-1}.$$

  - Conditional median:

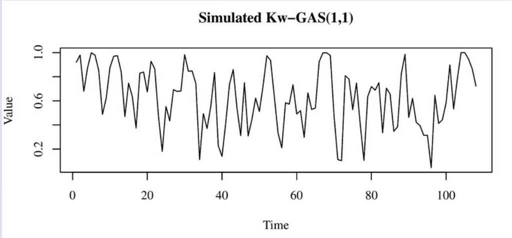
$$\mu_t = g^{-1}\left(f_t\right) = g^{-1}\left(\omega + \sum A_i s_{t-i} + \sum B_j f_{t-j}\right).$$

Scaled function of the past information: $s_t = S_t \nabla_t,$ where

$$\nabla_t = \frac{\varphi \mu_t^{\varphi-1}}{\left(1-\mu_t^{\varphi}\right)\log\left(1-\mu_t^{\varphi}\right)}\left(\frac{\log(0.5)}{\log\left(1-\mu_t^{\varphi}\right)}\log\left(1-y_t^{\varphi}\right)+1\right)\frac{1}{g'\left(\mu_t\right)},$$

and $$S_t = \frac{\left(1-\mu_t^{\varphi}\right)^2 \log^2\left(1-\mu_t \varphi\right)}{\varphi^2 \mu_t^{2\varphi-2}\left[g'\left(\mu_t\right)\right]^2}.$$

Figure 1 – A Kw-GAS (1,1) process with ω = 0.66, A = 0.04, B = 0.07, φ = 2.24, and n = 108



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Results and Conclusions

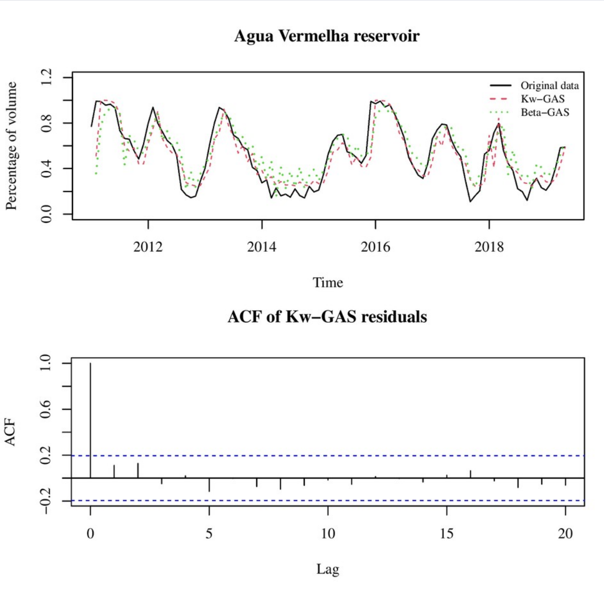
- We present a case study of 20 reservoirs from monthly percentage of useful volume of the main hydroelectric plant’s water reservoirs in the Southeast/Midwest subsystem of the Brazilian National Interconnected System.

Table 1 - Times (percentage in parenthesis) that each model presented the best fit in each resevoir.

	Measure	Kw-GAS	Beta-GAS
In-sample results	RMSE	14 (70%)	6 (30%)
	MAPE	13 (65%)	7 (35%)
	AIC	12 (60%)	8 (40%)
Out-of-sample forecasts*	RMSE	11 (55%)	9 (45%)
	MAPE	9 (45%)	11 (55%)

* Out-of-sample forecasts 1-step ahead, updated by actual observations.

Figure 2 - Overview of Time Series plot and ACF for one of the reservoirs database (Agua Vermelha).



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