

A novel observation-driven model for the conditional median of double-bounded time series

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Abbreviated abstract: A new generalized autoregressive score (GAS) model is introduced. It is defined from the assumption that the conditional median of the Kumaraswamy distribution is a time-varying parameter under the GAS framework and pioneers the conditional median approach to analyze double-bounded time series from the GAS specification. We conduct simulation studies and present an empirical application.

Related publications:

- D. Creal *et al*, Journal of Applied Econometrics 28 (5), 777-795 (2013)
- P. Gorgi *et al*, Journal of Econometrics In Press, (2021)



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The GAS Framework and Previous Works

- Introduced by Creal *et al* (2013), the GAS framework considers a sequence of random variables of interest, $\tilde{y}_1, \dots, \tilde{y}_n$, each one conditionally distributed as

$$\tilde{y}_t \sim p(\tilde{y}_t \mid f_t, F_{t-1}; \theta), t=1, \dots, n,$$

- The updating equation for f_t is

$$f_t = \omega + \sum A_i s_{t-i} + \sum B_j f_{t-j},$$

- The updating mechanism considers that $s_t = S_t \nabla_t$, where $\nabla_t = \partial \log(p(y_t \mid f_t, F_{t-1}; \theta) / \partial f_t)$ is the score function corresponding to f_t and S_t is a scale factor, which is usually taken from the inverse of the information matrix corresponding to the updating mechanism.

www.gasmodel.com/gaspapers.htm

Generalized Autoregressive Score models

The following articles, papers, and books are related to the GAS model (alphabetic order per year).

Download the main GAS Journal of Applied Econometrics paper (2013, open access) by clicking here!

If you find your contribution is missing, please let us know by sending an email to Andre Lucas (a.lucas@vu.nl). Before using any of the code, please read the disclaimer.

in press The GAS beta is the only option in the unit interval

228. Buccheri, Giuseppe, and Fulvio Corsi (in press): "Hark the Shark: Realized Volatility Modelling with Measurement Errors and Nonlinear Dependencies", *Journal of Financial Econometrics*.
Click here to download the working paper version.

227. Caballero, Diego, Andre Lucas, Bernd Schwaab, and Xin Zhang (in press): "Risk endogeneity at the lender/investor-of-last-resort", *Journal of Monetary Economics*.
Click here to download the ECB working paper version.

226. Catania, Leopoldo (in press): "Dynamic Adaptive Mixture Models with an Application to Volatility and Risk", *Journal of Financial Econometrics*.
Click here to download the working paper version.

Kumaraswamy autoregressive score model (Kw-GAS)

- The Kw-GAS model is defined from the assumption that the conditional median of the Kw distribution is a time-varying parameter under the GAS framework.
- Thus, the novel observation-driven model for the conditional median of double-bounded time series is fully specified by the following equations:

- Density:

$$p(\tilde{y}_t \vee f_t, F_{t-1}; \theta) = \frac{\varphi \log(0.5)}{\log(1 - \mu_t^\varphi)} y_t^{\varphi-1} (1 - y_t^\varphi)^{\log(0.5)/\log(1 - \mu_t^\varphi) - 1}.$$

- Conditional median:

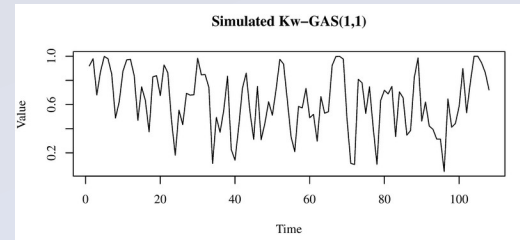
$$\mu_t = g^{-1}(f_t) = g^{-1}\left(\omega + \sum A_i s_{t-i} + \sum B_j f_{t-j}\right).$$

Scaled function of the past information: $s_t = S_t \nabla_t$, where

$$\nabla_t = \frac{\varphi \mu_t^{\varphi-1}}{(1 - \mu_t^\varphi) \log(1 - \mu_t^\varphi)} \left(\frac{\log(0.5)}{\log(1 - \mu_t^\varphi)} \log(1 - y_t^\varphi) + 1 \right) \frac{1}{g'(\mu_t)},$$

$$\text{and } S_t = \frac{(1 - \mu_t^\varphi)^2 \log^2(1 - \mu_t^\varphi)}{\varphi^2 \mu_t^{2\varphi-2} [g'(\mu_t)]^2}.$$

Figure 1 – A Kw-GAS (1,1) process with $\omega = 0.66$, $A = 0.04$, $B = 0.07$, $\varphi = 2.24$, and $n = 108$



Results and Conclusions

- We present a case study of 20 reservoirs from monthly percentage of useful volume of the main hydroelectric plant's water reservoirs in the Southeast/Midwest subsystem of the Brazilian National Interconnected System.

Table 1 - Times (percentage in parenthesis) that each model presented the best fit in each reservoir.

| | Measure | Kw-GAS | Beta-GAS |
|--------------------------|---------|----------|----------|
| In-sample results | RMSE | 14 (70%) | 6 (30%) |
| | MAPE | 13 (65%) | 7 (35%) |
| | AIC | 12 (60%) | 8 (40%) |
| Out-of-sample forecasts* | RMSE | 11 (55%) | 9 (45%) |
| | MAPE | 9 (45%) | 11 (55%) |

* Out-of-sample forecasts 1-step ahead, updated by actual observations.

Figure 2 - Overview of Time Series plot and ACF for one of the reservoirs database (Agua Vermelha).

