Asymmetric prior in wavelet shrinkage

Alex Rodrigo dos Santos Sousa
Institute of Mathematics and Statistics, University of São Paulo

Abbreviated abstract:

This presentation proposes the use of an asymmetric shrinkage rule based on the discrete mixture of a point mass function at zero and an asymmetric beta distribution as prior to the wavelet coefficients in a nonparametric regression model.

Related publications:

– Sousa, A.R.S., arXiv:2010.04666 (2020)
– Sousa, A., Garcia, N. and Vidakovic, B., Computational Statistics (2020)

3rd Conference on Statistics and Data Science
Salvador, Brazil (online)
October 28-30, 2021

alex.sousa89@usp.br

Statistical model

- We consider the nonparametric model

$$y_i = f(x_i) + e_i \quad , \qquad i = 1, \dots, n = 2^J$$

where f is unknown, (x_i, y_i) are data points and $e_i \sim N(0, \sigma^2)$.

- The goal is to estimate the unknown function f by representing it as wavelet basis expansion

$$f(x) = \sum_{j,k \in \mathbb{Z}} \theta_{jk} \psi_{jk}(x).$$

- Empirical (noisy) wavelet coefficients d are obtained by application of a discrete wavelet transform (DWT), represented by its orthogonal transformation matrix $\boldsymbol{W}$, on the data, which gives us the model in the wavelet domain

$$\boldsymbol{d} = \boldsymbol{\theta} + \boldsymbol{\varepsilon}$$

where $\boldsymbol{d} = \boldsymbol{W}\boldsymbol{y}, \boldsymbol{\theta} = \boldsymbol{W}\boldsymbol{f}$ and $\boldsymbol{\varepsilon} = \boldsymbol{W}\boldsymbol{e} \sim N(\boldsymbol{0}, \sigma^2 \boldsymbol{I})$.

alex.sousa89@usp.br

3rd Conference on
Statistics and
Data Science
Salvador, Brazil (online)
October 28-30, 2021

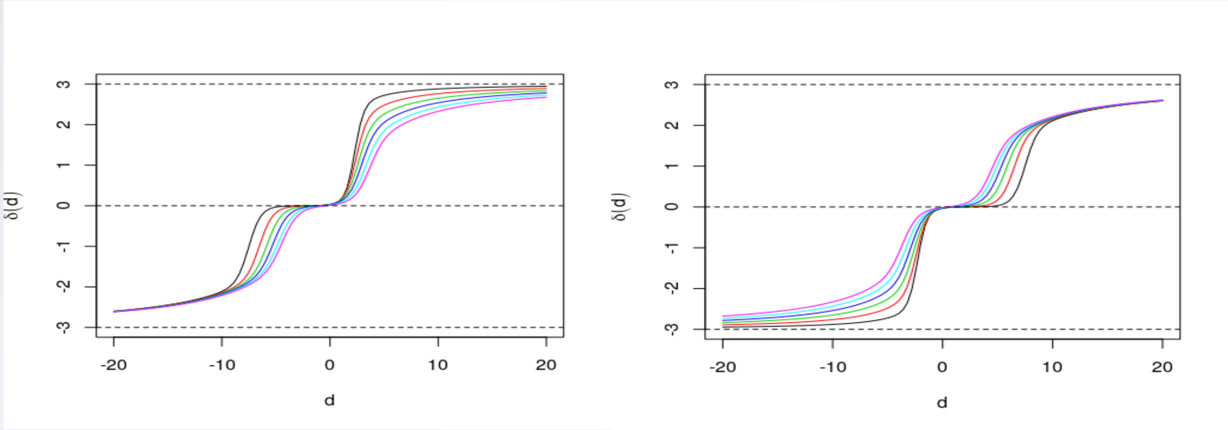
Bayesian shrinkage rule

- We propose the following asymmetric prior distribution to a single wavelet coefficient θ:

$$\pi(\theta) = \alpha\delta_0(\theta) + (1-\alpha)\frac{(\theta+m)^{a-1}(m-\theta)^{b-1}}{(2m)^{a+b-1}B(a,b)} \quad , \qquad \theta \in [-m, m]$$

where $\delta_0(.)$ is a point mass function at zero and $\alpha \in (0,1)$, $a, b, m > 0$ are hyperparameters.

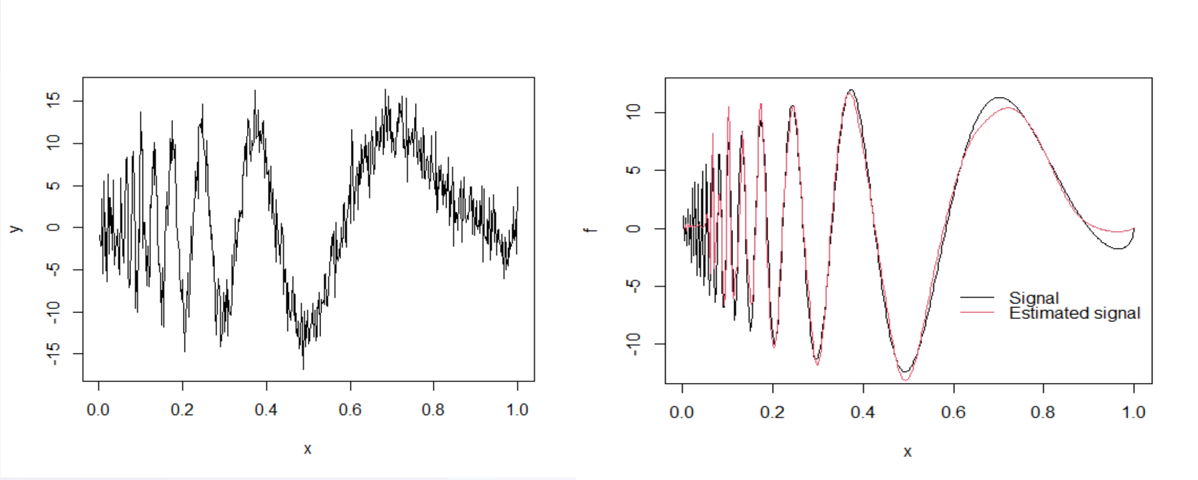
- Under squared error loss function, the associated shrinkage rule is given by $\delta(\theta) = E_\pi(\theta|d)$.



alex.sousa89@usp.br

Results and Conclusions

- The shrinkage rule under the proposed prior had great performance in terms of AMSE and AMAE measures against standard shrinkage/thresholding techniques in two simulation studies involving asymmetrically distributed wavelet coefficients estimation (1) and the so called Donoho-Johnstone test functions (2).



alex.sousa89@usp.br

3rd Conference on
Statistics and
Data Science
Salvador, Brazil (online)
October 28-30, 2021