Testing the Granger-causal relationship between the North Atlantic Oscillation and multidecadal sea surface temperature variability

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Abbreviated abstract: We explore the possibility of using high-resolution paleoclimate archives to test Granger causality between the North Atlantic Oscillation (NAO) and sea surface temperature (SST). We do this by identifying the lead/lag relationships using sieve-bootstrapped cross-correlation functions and vector autoregression (VAR) models with restricted near-contemporaneous lags. Further, we obtain bootstrapped distributions of three out-of-sample statistics for testing Granger causality using the VARs. Finally, we investigate stronger signals by clustering the paleoclimate time series while accounting for time uncertainty in these records, using dynamic time warping.

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Problem, Data, Previous Works

Wills et al. (2018) find evidence in models and instrumental observations that AMOC and NAO are coupled on decadal to multidecadal timescales.

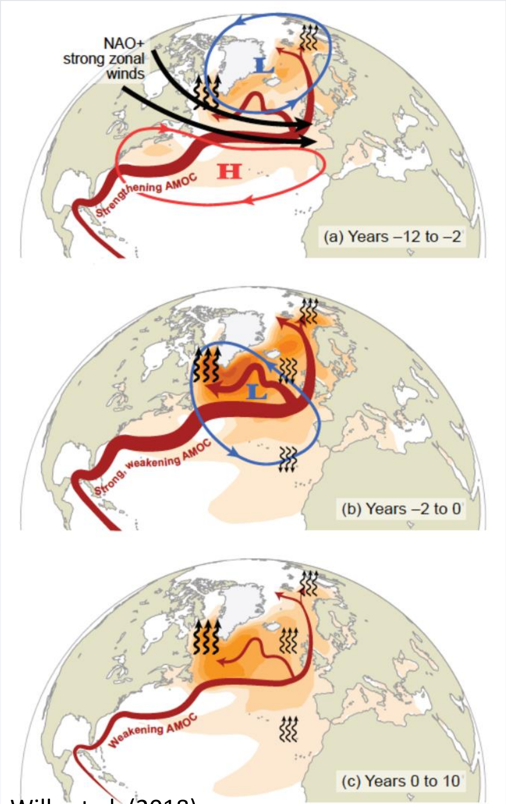
We use existing *paleoclimate data* to look for the expected lead-lag relationships between reconstructed NAO and SSTs from high and low latitude sites. We also take the analysis one step further by conducting Granger-type causality tests on these data.

Data selection criteria

- At least four dating constraints in the last 2000 years
- Lowest resolution of 100 years/sample on average over the last 2000 years
- Records must be longer than 100 years

Data challenges

- different temporal resolutions
- different lengths
- unequally spaced
- time uncertainty
- non-stationary

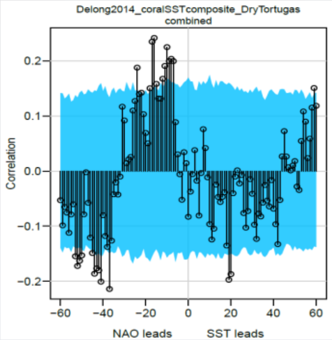


Wills et al. (2018)

Methods

1) 200-year loess detrending (stats::loess)

2) Cross-correlation with sieve bootstrap (funtimes::ccf_boot)



3) Restricted VARs (vars::VAR, GitHub.com/vlyubchich/vars)

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \sum_{j=1}^{k} \begin{bmatrix} \alpha_{11,j} & \alpha_{12,j} \\ \alpha_{21,j} & \alpha_{22,j} \end{bmatrix} \begin{bmatrix} y_{t-j} \\ x_{t-j} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} + \sum_{j=1}^{k} \begin{bmatrix} \beta_{11,j} & 0 \\ \beta_{21,j} & \beta_{22,j} \end{bmatrix} \begin{bmatrix} y_{t-j} \\ x_{t-j} \end{bmatrix} + \begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix}$$

$$\alpha_{12,j} = 0 \text{ and } \alpha_{21,j} = \beta_{21,j} = 0 \text{ for } j = 1, \ldots, 10$$

exclude near-contemporaneous dependencies between NAO and SST; $\max(k) = 10\log_{10} T/2$

4) Testing of Granger causality of *x* using out-of-sample (′) errors

$$H_0 : \mathrm{E}(\varepsilon_{1t}^{\prime 2}) = \mathrm{E}(w_{1t}^{\prime 2}) \qquad H_1 : \mathrm{E}(\varepsilon_{1t}^{\prime 2}) < \mathrm{E}(w_{1t}^{\prime 2})$$

and bootstrapped versions of statistics

$$d_t = \varepsilon_{1t}^{\prime 2} - w_{1t}^{\prime 2}$$

$$d_t = a + u_t \qquad (\varepsilon_{1t}' - w_{1t}') = c\,(\varepsilon_{1t}' + w_{1t}') + u_t$$

$$\mathrm{MSEt} = \frac{\hat{a}}{se(\hat{a})} \qquad \mathrm{MSEcor} = c$$

5) Clustering SST loess residuals using dynamic time warping, finding optimal clustering strategy and number of clusters using Dunn and Silhouette indices, repeating steps 2–4 on NAO and cluster centroids of SSTs

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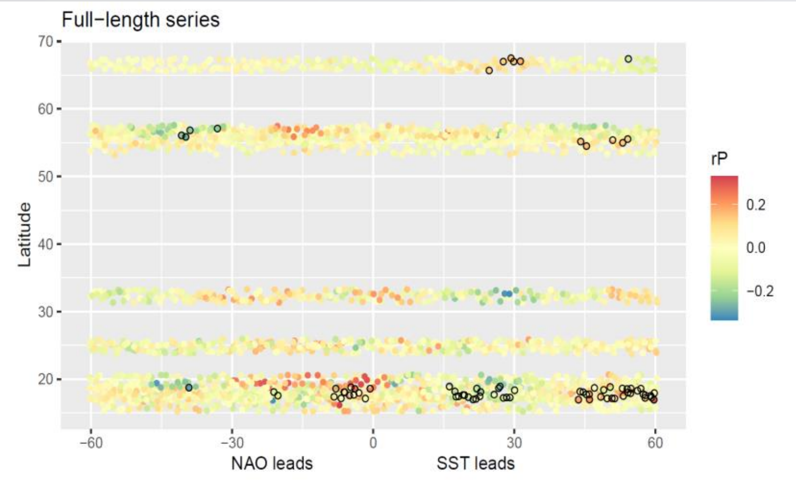
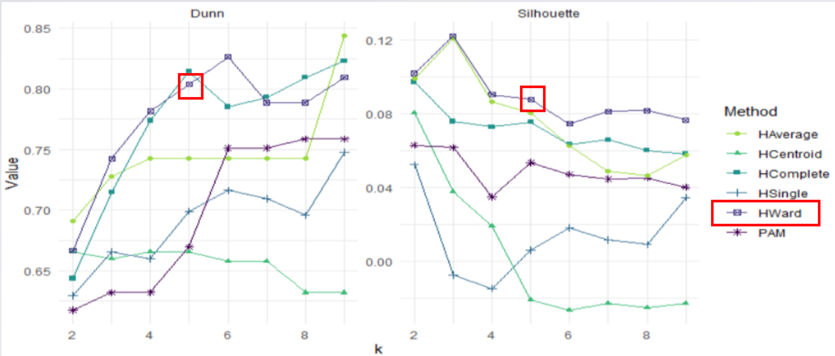
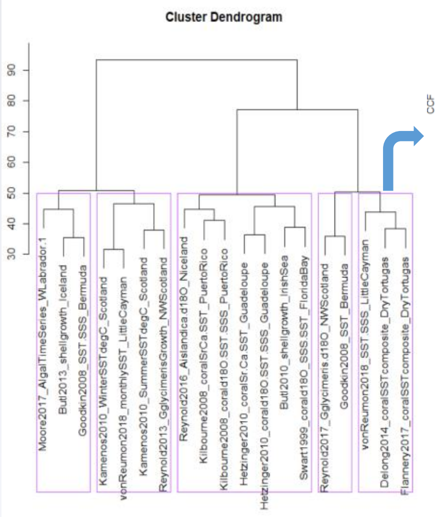
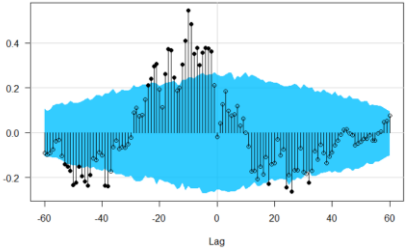
Results and Conclusions

1 NAO time series, 21 time series of SST proxies









Circles denote significance after Benjamini–Hochberg adjustments to keep FDR at 5%

Both MSEt & MSEcor detected NAO cause of SST in clusters 1 and 3 (at k = 11). No reverse links detected.

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