

# Variational Approach of Motor Control

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**Abbreviated abstract:** The dynamics that govern the temporal evolution of the trajectory of the upper limbs has been derived from the minimum jerk principle and has been experimentally confirmed. This work proposes another organizational principle for the control and execution of movement and compares which model better fits the experimental data. The motor control theory proposed in this work is based on parameter optimization methods, through variational calculus and optimal control theory. The cost function to be minimized is the integral of time of the square of acceleration.

## **Related publications:**

- MIRANDA, J.G.V. et al., *Scientific Reports* 8 (1), 12918 (2018)
- FLASH, T.; HOGAN, *The Journal of Neuroscience* 5 (7), 16688-1703 (1985)

# Previous Works

## Minimum Jerk Model

- (Hogan, 1984)
- (Flash; Hogan, 1985)

## Minimum Jerk Model Plus Time Cost

- (Hoff, 1994)

## Movement Element Decomposition

- (MIRANDA, J.G.V. et al., 2018)

Reach



$$A = \int_{t_0}^{t_f} \left( \left( \frac{d^3x}{dt^3} \right)^2 + \left( \frac{d^3y}{dt^3} \right)^2 \right) dt.$$

Reach



$$q(t) = q(t_f) \left( 6 \left( \frac{t}{t_f} \right)^5 - 15 \left( \frac{t}{t_f} \right)^4 + 10 \left( \frac{t}{t_f} \right)^3 \right).$$

Reach

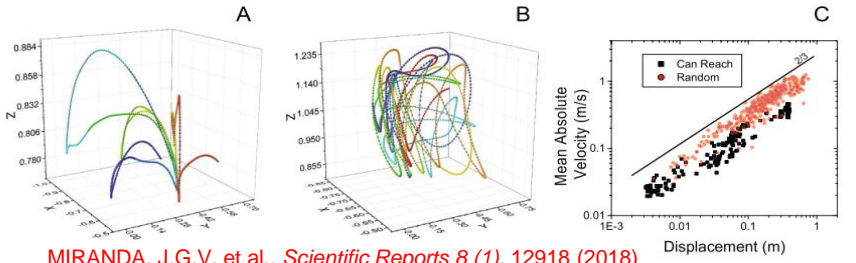


$$t_f = (60D)^{1/3} R^{1/6}.$$

Random



$$v_m = \frac{D^{2/3}}{60^{1/6} K^{1/6}},$$



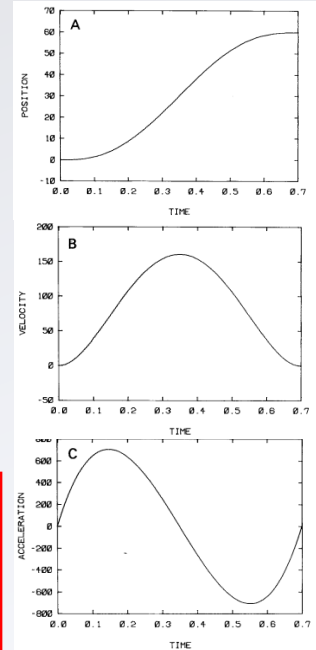
MIRANDA, J.G.V. et al., *Scientific Reports* 8 (1), 12918 (2018)

## Our Proposal

### Minimum Acceleration Model

$$A = \int_{t_0}^{t_f} \left( \left( \frac{d^2x}{dt^2} \right)^2 + \left( \frac{d^2y}{dt^2} \right)^2 \right) dt.$$

$$q(t) = q(t_f) \left( -2 \left( \frac{t}{t_f} \right)^3 + 3 \left( \frac{t}{t_f} \right)^2 \right).$$



HOGAN., *Journal of Neuroscience* 4 (11), 2745-2574 (1984)



# Methods

## Participants:

- Ten healthy subjects
- Seven males, Nine right-handed

## Experimental Procedures:

- Ten camera
- Tree-dimensional random
- One-dimensional goal-directed movements

## Data Processing:

- Low-Pass Filter (10 Hz)
- Derived Velocity Profile
- Movement Element Decomposition (MED)

## Statistical:

- Descriptive Statistics
- Pearson Correlation
- Estimator W
- Wilcoxon Teste

$$W = \frac{1}{N} \sum_{i=1}^N \frac{\sigma(w_{i,m}(t) - v_{i,m}(t))}{\overline{w_{i,m}}}$$

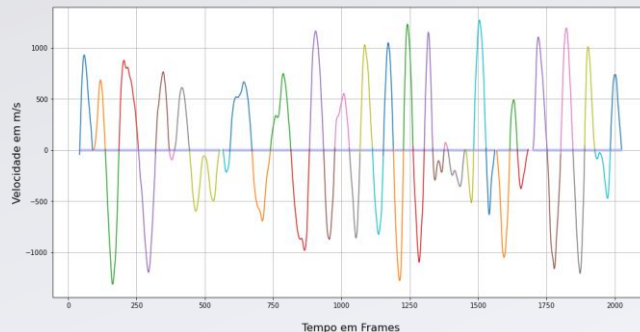
## Random:

LimT = 100 ms  
LimD = 3mm  
LimV = 10 mm/s

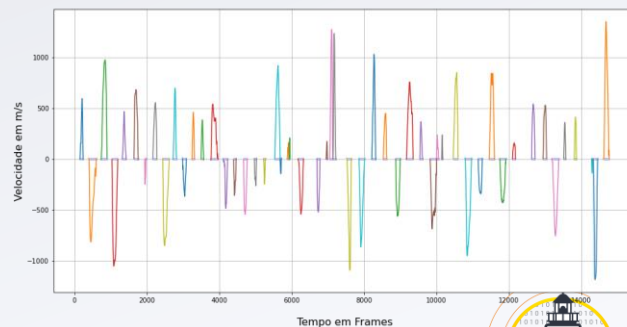
## Reach:

LimT = 200 ms  
LimD = 20 mm  
LimV = 10 mm/s

Perfil de Velocidade



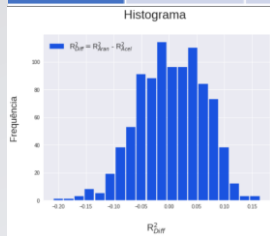
Perfil de Velocidade



# Results and Conclusions

## Reach

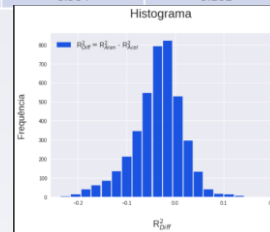
| Estatística Descritiva |            |     |       |         |               |
|------------------------|------------|-----|-------|---------|---------------|
| $R^2$                  | Modelo     | N   | Média | Mediana | Desvio Padrão |
|                        | Arranque   | 936 | 0.809 | 0.862   | 0.178         |
|                        | Aceleração | 936 | 0.805 | 0.858   | 0.161         |



Shapiro-Wilk: p-value < 0.001  
 Kolmogorov-Smirnov: p-value = 0.036  
 Wilcoxon: p-value = 0.002

## Random

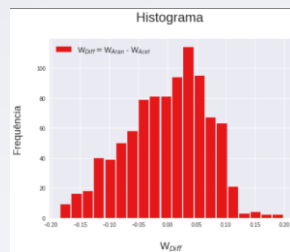
| Estatística Descritiva |            |      |       |         |               |
|------------------------|------------|------|-------|---------|---------------|
| $R^2$                  | Modelo     | N    | Média | Mediana | Desvio Padrão |
|                        | Arranque   | 4084 | 0.752 | 0.872   | 0.276         |
|                        | Aceleração | 4084 | 0.787 | 0.904   | 0.262         |



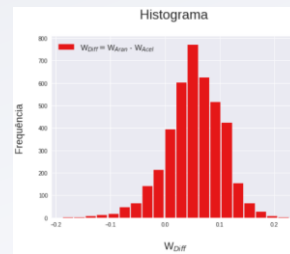
Shapiro-Wilk: p-value < 0.001  
 Kolmogorov-Smirnov: p-value < 0.001  
 Wilcoxon: p-value = 0.002

| Estatística Descritiva |            |     |       |         |               |
|------------------------|------------|-----|-------|---------|---------------|
| $W$                    | Modelo     | N   | Média | Mediana | Desvio Padrão |
|                        | Arranque   | 936 | 0.287 | 0.239   | 0.184         |
|                        | Aceleração | 936 | 0.298 | 0.251   | 0.164         |

Shapiro-Wilk: p-value < 0.001  
 Kolmogorov-Smirnov: p-value = 0.03  
 Wilcoxon: p-value = 0.86



| Estatística Descritiva |            |      |       |         |               |
|------------------------|------------|------|-------|---------|---------------|
| $W$                    | Modelo     | N    | Média | Mediana | Desvio Padrão |
|                        | Arranque   | 4084 | 0.307 | 0.217   | 0.249         |
|                        | Aceleração | 4084 | 0.253 | 0.175   | 0.226         |



Shapiro-Wilk: p-value < 0.001  
 Kolmogorov-Smirnov: p-value < 0.001  
 Wilcoxon: p-value = 0.002

